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POSSIBILISTIC PROGRAMMING IN PORTFOLIO SELECTION

ABSTRACT: The paper discusses the formulation of a possibilistic programming model aimed at selecting the optimal portfolio. The author proposes possibilistic distributions for returns and variances of stock prices that are not traded daily. In addition to maximizing expected returns while minimizing risk, the model also incorporates several indicators calculated based on financial reports: paid dividends, liquidity, efficiency of physical and intellectual capital. An illustrative example based on data from the Belgrade Stock Exchange is presented.

KEY WORDS: portfolio, optimization, fuzzy sets.

1. Introducion

The problem of optimization is ubiquitous in numerous areas of human activity, particularly in the fields of economics and especially finance, see for example Crave and Sardar (2005), Hirschey (2009) and Luptáčik (2010). The primary goal of optimization is to search for the most favorable among the available alternative solutions to various problems, within the constraints set. In economics, common problems for which optimized solutions are sought include, among others, costs,

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product and service quality, production, and investment portfolios. The aim is to achieve maximum profitability under the existing constraints that is sustainable in the long term.

The primary goal of an investor is to maximize returns while minimizing risk. However, this goal is difficult to achieve given the interdependence of risk and return, which is commonly measured by the Sharpe ratio (see, for example, Njegomir (2011), Kapil (2011), Brigham and Houston (2012)). Nobel laureate Markowitz (1952) was the first to theoretically demonstrate that forming a portfolio, as opposed to selecting individual attractive stocks, can better reduce investment risk. He was also the first to highlight the necessity of a trade-off between risk and return in a portfolio. Optimization in investing represents the effort of investors to create investment portfolios that will enable them to maximize returns for a given level of investment risk, or to minimize risk with the constraint of achieving a desired level of expected return (see, for example, Jones (2010) and Anderson et al. (2012)).

Following Markowitz's work, the problem of selecting an optimal portfolio has shifted towards relaxing the initial assumptions. There are two main groups of methods for solving the optimization problem: probabilistic and possibilistic (fuzzy) programming. Problems are further framed as either single-criteria (minimizing risk with fixed levels of return and forming an efficient frontier) or multi-criteria (minimizing risk while maximizing returns, maximizing dividend payments, etc.). In this paper, the method of multi-criteria fuzzy programming will be applied to solve the problem of selecting the optimal portfolio. We are interested in the possibilities and limitations of investing in companies from Vojvodina. The list of companies was taken from the website of the Serbian Business Registers Agency, and the stock prices of individual companies were sourced from the Belgrade Stock Exchange website.

2. Literature review

Although probability theory is the main tool for analysing uncertainty in finance, the market is also influenced by factors that are not stochastic in nature. These include linguistic descriptions of financial variables, which are characterized by two types of uncertainty: ambiguity, such as "a return of around 12%," and vagueness, in terms of setting clear boundaries, such as "high risk." Fuzzy mathematical programming emerged from the need to adequately solve optimization problems that involve such uncertainties. The following is a brief review of the literature that has used fuzzy methodology to solve the optimal portfolio problem. For a detailed review of the literature on fuzzy programming, see Figueroa-García et al. (2022).

Lin and Liu (2008) present three possible models for portfolio selection with minimum quantities of purchased stocks and develop corresponding genetic algorithms (GA) to obtain solutions. The results show that the portfolios obtained in this way are very close to the efficient frontier, indicating that the proposed methodology can produce near-optimal and feasible solutions in real time. Vercher et al. (2007) propose two models for portfolio selection aimed at minimizing "downside" risk, i.e., semi-variance, with the constraint that the return must not fall below a predetermined value. Returns on individual assets are approximated using LR fuzzy numbers of the same shape, while expected return and risk are evaluated using interval means. Vercher (2008) further explores new models to solve the previously stated problem and proposes SIP (semi-infinite programming) with relaxed constraints.

Gupta et al. (2008) use fuzzy methodology to assess expected returns, liquidity, and risk. Fuzzy methodology allows the inclusion of subjective characteristics in the portfolio selection model, providing a basis for expressing individual investor preferences. Wei Guo Zhang et al. (2007) consider the problem of selecting portfolios of assets defined by upper and lower possibilistic means and variances. The authors transform the MV model into a linear one using possibilistic distributions, making their approach suitable for solving the problem of selecting "large" portfolios. The authors propose two portfolio selection models and introduce the concept of lower and upper possibilistically efficient portfolios. Zhang et al. (2010) use credibility theory to develop a model for adjusting an existing portfolio to account for transaction costs. Returns are modelled with triangular fuzzy numbers, and sequential quadratic programming is applied to obtain the optimal strategy. Zhang et al. (2011) further build on the issue raised in the previous work, now from the perspective of possibilistic MV (mean-variance) theory. They propose a portfolio optimization model using a V-shaped transaction cost function to transition from the current portfolio to the adjusted one.

Huang (2008) addresses the portfolio selection problem using semi variance (SV) as a fuzzy variable and proves certain properties of fuzzy semi variances. Bhattacharyya et al. (2011) propose two MSV (mean-semi-variance) models, using interval estimates and considering the impact of transaction costs in an MV model with asymmetry. Results from numerical experiments show that the proposed algorithm is effective in solving fuzzy MSV models.

Li et al. (2010) observe asymmetry in portfolio return distributions, noting that for equal values of expected returns and variances, investors prefer portfolios with greater asymmetry. The authors define an asymmetry coefficient for fuzzy variables and examine its properties, proposing an extension of the fuzzy MV model—the MV model with asymmetry. In solving the problem, they construct genetic algorithms (GA) to integrate fuzzy simulations. Bhattacharyya et al. (2011) address a similar problem, proposing three models solved by integrating fuzzy simulation (FS) with GA to create a powerful hybrid intelligent algorithm (HIA).

Tanaka and Guo (1999) identify two types of possibilistic distributions—upper and lower distributions—used to evaluate expert opinions in portfolio selection. The portfolio selection is formulated as a quadratic problem. The authors conclude that portfolio returns measured using the lower possibilistic distribution have a narrower range than returns obtained from the upper possibilistic distribution.

Kocadagli and Keskin (2015) introduce a new portfolio selection model that considers risk preferences in line with market changes. Thakur et al. (2018) apply the fuzzy Delphi method to identify weakly correlated factors that indirectly affect the market, while Brito (2023) uses a utility, entropy, and variance (EU-EV) model to preselect stocks, followed by the application of the classic MV model. Finally, Savaei et al. (2024) propose a solution to the portfolio optimization problem for investors who are risk averse.

3. Research and methodology

The rationale for using a multi-criteria portfolio selection model lies in the fact that expected return and risk do not encompass all the information necessary for making an investment decision. By incorporating additional criteria, it is possible to modify decisions, selecting portfolios that may not dominate in an MV (mean-variance) environment but compensate with excellent performance on other criteria, thus becoming dominant in a multi-criteria framework. Following the work of Gupta et al. (2008), the criteria we use are the following: short-term returns, long-term returns, risk, dividends, liquidity, efficiency of intellectual capital, and efficiency of physical capital. The last two criteria utilize fundamental indicators from the financial reports of the observed companies, and it is interesting to see how these additional criteria influence the adjustment of the optimal choice.

In the problem setup of this paper, investments in risk-free assets are also allowed. We use the following notations:

- r_{f} return on the risk-free instrument,
- r_i return on the stock of the i-th company, where i=1,2,...,n,
- x_i the proportion of total funds invested in the i-th stock,
- l_i liquidity of the i-th stock,
- d_i annual dividend on the i-th stock.

Now, we will formulate the objectives (criteria) and constraints: Returns on risky assets are modelled with trapezoidal fuzzy numbers, which emphasize the uncertainty of the financial market and the imprecise and incomplete data available. For a more detailed insight into fuzzy set theory, see the foundational works of Zadeh (1965) and Dubois and Prade (1980, 1987). A fuzzy number $A=(a,b,\alpha,\beta)$ is a trapezoidal fuzzy number if its membership function is given by:

$$\mu_{A}(x) = \begin{cases} 1 - \frac{a - x}{\alpha}, a - \alpha \le x \le \alpha \\ 1, a \le x \le b \\ 1 - \frac{x - b}{\beta}, b \le x \le b + \beta \end{cases}$$

Let $r_i = (a_i, b_i, \alpha_i, \beta_i)$ return on the i-th stock from the portfolio. For a portfolio with n risky and one risk-free asset $\mathbf{x} = (x_p, x_1, ..., x_n)$, the fuzzy return is given with:

 $\Pi_{s}(\mathbf{x}) = \mathbf{r}_{f}\mathbf{x}_{f} + \mathbf{r}_{1}\mathbf{x}_{1} + \dots + \mathbf{r}_{n}\mathbf{x}_{n} = (\Sigma \mathbf{a}_{j}\mathbf{x}_{j}, \Sigma \mathbf{b}_{j}\mathbf{x}_{j}, \Sigma \mathbf{a}_{j}\mathbf{x}_{j}\Sigma \beta_{j}\mathbf{x}_{j})$ $= (\mathbf{A}(\mathbf{x}), \mathbf{B}(\mathbf{x}), \mathbf{\alpha}(\mathbf{x}), \mathbf{\beta}(\mathbf{x})).$

Figure 1 Membership finction of the trapezoidal fuzzy number

The expected value of the return on the portfolio Π given by the trapezoidal fuzzy number was defined by Dubois and Prade (1987)² as an interval [E_i, E^{*}] = [A(**x**) - $\alpha(\mathbf{x})/2$, B(**x**) + $\beta(\mathbf{x})/2$]. By defuzzification, we get the arithmetic mean of this interval, E(Π) = $\Sigma \frac{1}{2}$ [$\mathbf{a}_i + \mathbf{b}_i + \frac{1}{2}$ ($\beta_i - \alpha_i$)] \mathbf{x}_i , as an estimate of the expected return on the portfolio Π that we maximize. The choice of specific values of a, b, α and β , as well as the shape of the function belonging to the phase number to describe the return on each stock are somewhat arbitrary.

The annual portfolio dividend is calculated as:

 $D(\mathbf{x}) = d_1 x_1 + d_2 x_2 + \dots + d_n x_n.$

As an approximation for the amount of dividends, we use relative earnings per share (EPS).

Portfolio risk is measured by the semi-absolute deviation of the return on portfolio x below the expected return. It is considered that devi-

² The lower and upper bounds of the expected return given with the fuzzy number r are defined as $E_*(r) = {}_0\int^1 (\inf r_{\alpha}) d\alpha$ i $E^*(r) = {}_0\int^1 (\sup r_{\alpha}) d\alpha$, where $\inf r_{\alpha}$ and $\sup r_{\alpha}$ denote the extreme points of the α -cut of the set r.

ations above the expected return cannot be treated as risky, but desirable, and they do not affect the level of risk. The "bottom risk" measure describes investors' preferences in a more realistic way because it penalizes only negative deviations from the expected return. Stevenson (2001) indicates the correctness of using the "bottom risk" measure in the case of emerging markets where returns are not normally distributed. The mean semi-absolute deviation of returns on portfolio x was proposed by Speranza (1993) with the formula: $E\left(\min\left\{0, \sum r_j x_j - E\left(\sum r_j x_j\right)\right\}\right)$.

For working with trapezoidal fuzzy returns, the following formula for the semi-deviation of portfolio returns, analogous to Speranza's formula, is more suitable: σ (Π) = E(max{0, E(Π) - Π }).

It is easy to show that the interval in which the semi-deviation is located has the following form:

 $\sigma(\Pi) = [0, B(\mathbf{x}) - A(\mathbf{x}) + \frac{1}{2}(\alpha(\mathbf{x}) + \beta(\mathbf{x}))], \text{ that is, after defuzzification:} \\ \sigma(\Pi) = \sum \frac{1}{2} [b_i - a_i + \frac{1}{2}(\beta_i + \alpha_i)] x_i.$

Constraints

Taking into account that the largest number of shares of Vojvodina companies are traded according to the prevailing price method, and that trades are infrequent, the liquidity coefficient for each share is measured by the share of days during which trade was carried out in the entire observed period, l(ai) li = ti/T, and portfolio liquidity is a linear combination of individual liquidity: $L(x) = x_f + l_1x_1 + ... + l_nx_n$.

Budget constraint:

 $x_f + x_1 + \dots + x_n = 1.$

Maximum (minimum) share of capital invested in an individual share:

 $l_i \le x_i \le u_i, i = 1, 2, ..., n.$

Minimum share of capital invested in a risk-free asset:

 $\mathbf{x}_{f} \ge \mathbf{l}_{f}$

The maximum and minimum share of capital depend on several fundamental factors, e.g. industry trends, minimum number of shares that must be purchased, small business capitalization, etc. Intellectual capital efficiency is calculated as the sum of human capital efficiency (HCE) and structural capital efficiency (SCE). Lee (2010) defines the efficiency of human capital as the proportion of information that has been formalized. Without diminishing the importance of Lee's theoretical approach, he opts for the previously introduced definition that uses available data from financial statements. HCE measures how much value added (VA)³ is created for each monetary unit invested in employees. Structural capital (SC)⁴ represents the result of the work of human capital in the past, and its efficiency is reflected in the share in the added value created:

 $ICE_i = VA_i/BPZ_i + SC_i/VA_i$. We expect that firms that are highly efficient (ICE > 2.5) generate additional returns more easily (Pulic (2003)). The intellectual capital efficiency of portfolio x is given by:

$$ICE(\mathbf{x}) = ICE_1\mathbf{x}_1 + ICE_2\mathbf{x}_2 + \dots + ICE_n\mathbf{x}_n.$$

Intellectual capital produces value in cooperation with physical and financial capital. Efficiency of use of physical capital, CEE represents the share of added value in the total assets of the company (TA):

CEE = VA/TA. CEE shows how much added value is created for each monetary unit invested in physical capital. The efficiency of physical capital in portfolio x is given by:

 $CEE(\mathbf{x}) = CEE_1\mathbf{x}_1 + CEE_2\mathbf{x}_2 + \dots + CEE_n\mathbf{x}_n.$

Based on the previously stated objectives and limitations, it is possible to formulate the problem of choosing the optimal portfolio as follows:

$$\begin{split} & \text{Max } \Pi_{s}(\mathbf{x}) = r_{t}x_{f} + r_{1}x_{1} + ... + r_{n}x_{n} \\ & \text{Min } \sigma'(\Pi) = \Sigma \frac{1}{2} [b_{i} - a_{i} + \frac{1}{2} (\beta_{i} + \alpha_{i})]x_{i} \\ & \text{Max } D(\mathbf{x}) = d_{1}x_{1} + d_{2}x_{2} + ... + d_{n}x_{n} \\ & \text{Max } L(\mathbf{x}) = x_{f} + l_{1}x_{1} + ... + l_{n}x_{n} \end{split}$$

³ The added value of the company is calculated according to the formula PBT + GSE + A + D (PBT = profit before taxation, GSE = gross salary of employees and other expenses, A = amortization, D = depreciation).

 $^{^{4}}$ SC = VA – HC

s. t. $x_f + x_1 + ... + x_n = 1$, $l_i \le x_i \le u_i$, i = 1, 2, ..., n, $x_f \ge l_f$, $ICE(\mathbf{x}) = ICE_1x_1 + ICE_2x_2 + ... + ICE_nx_n \ge ice_{min}$, $CEE(\mathbf{x}) = CEE_1x_1 + CEE_2x_2 + ... + CEE_nx_n \ge cee_{min}$, $x_i \ge 0$, i = 1, 2, ..., n. (short sales not permitted)

4. Results and discussion

Vojvodina companies whose (ordinary) shares are traded on the Belgrade Stock Exchange were selected as a numerical example. We have excluded those companies whose shares have not been traded at all in the last 3 years, leaving only 9 companies, given by sector: A: Agrobačka (1), Grupa Univereksport (2), Omoljica (3), Sloga Kać (4),

B: NIS (5),

C: Utva silosi (6),

M: Polj. Stručna služba Subotica (7),

N: Novosadski sajam (8), Revnost (9).

In Solving the optimization problem, the LINGO 20.0 application was used. Since the application is not able to solve the multi-criteria program, the so-called grid search method and solves the problem for different values of risk, dividend yield and liquidity parameters. The value of the risk variable is observed in the interval from 10% to 70%, with a step of 1 percentage point. At a fixed risk value, the optimization problem is solved for different values of dividend yield and liquidity, and in this way Pareto optimal solutions are obtained. Tables 1 and 2 show optimal portfolios for different levels of risk.

risk	share of each asset in the portfolio									
	2	3	4	5	6	7	8	9	rf	
0,25	0,081	0,026	0,25	0,247	0,25	-	0,047	-	0,1	
0,30	0,039	0,070	0,25	0,236	0,25	-	0,056	-	0,1	
0,35	-	0,113	0,246	0,225	0,25	-	0,065	-	0,1	
0,40	-	0,152	0,197	0,215	0,25	-	0,085	-	0,1	
0,45	-	0,191	0,148	0,206	0,25	-	0,105	-	0,1	
0,50	-	0,23	0,099	0,197	0,25	-	0,125	-	0,1	
0,55	-	0,25	0,074	0,191	0,25	-	0,129	0,006	0,1	
0,60	-	0,25	0,074	0,189	0,25		0,117	0,019	0,1	
0,65	-	0,25	0,075	0,188	0,25	-	0,106	0,032	0,1	
0,70	-	0,25	0,076	0,186	0,25	-	0,094	0,044	0,1	

Table 1: Overview of Pareto optimal solutions when limiting investment in individual funds to 25%

Authors' calculations

The investment in agriculture varies between 30% and 35%, and is carried out through investments in two companies across the full range of risk. All companies in the agriculture sector are characterized by low liquidity and a low intellectual capital coefficient. The mining sector is represented by only one company from Vojvodina (5), and it was included in all optimal portfolios, with its share decreasing as risk increases. It is characterized by high liquidity, a high intellectual capital coefficient, and dividend payouts. The manufacturing industry is also represented by one company (6), which is present in all portfolios with maximum participation. Professional, scientific, innovative, and technical activities are represented by one company (7), but due to its negative expected return, it was not selected for any of the portfolios. Finally, the administrative and support services sector is represented by two companies (8, 9), with relatively small and high expected returns, respectively.

Return	Dividends	Human Capital	Liquidity	Return on Assets
0,219	0,1	2,5	0,370	0,165
0,253	0,1	2,5	0,358	0,163
0,286	0,1	2,5	0,347	0,161
0,316	0,1	2,5	0,337	0,160
0,347	0,1	2,5	0,327	0,151
0,377	0,1	2,5	0,316	0,157
0,407	0,1	2,5	0,307	0,157
0,437	0,1	2,5	0,309	0,158
0,467	0,1	2,5	0,307	0,159
0,496	0,1	2,5	0,305	0,160

Table 2: Pareto optimal solutions in the case of limiting investment in
risky assets to 25%

Author's calculations

In Table 2, an overview of other parameters of the selected portfolios is provided: stock earnings (dividends), liquidity, and return on assets (efficiency of physical capital). All portfolios have the same level of intellectual capital efficiency, amounting to 2.778. In the formula for calculating the portfolio's intellectual capital efficiency, government bonds are excluded, hence the value is obtained through normalization⁵. The return potentially achieved through dividend payouts for all portfolios is at the required minimum of 0.1. The variance method generates portfolios with a higher coefficient of human capital efficiency compared to other methods used. Lower expected returns correspond to higher returns on assets, i.e., values of the physical capital efficiency coefficient, which align with the optimal portfolios obtained by maximizing the left and right return ranges, although on average, returns on assets are 1 percentage point higher for variance portfolios. Liquidity decreases with an increase in the desired return and corresponds to the inclusion of less liquid and simultaneously riskier stocks.

⁵ The value was obtained by normalizing the required lower limit of 2.5 by dividing it by the sum of the weights 0.9 of all risky assets.

5. Conclusion

This paper aims to highlights the integration of non-traditional factors like intellectual capital and liquidity into the portfolio optimization process, using possibilistic programming to handle limited data or fluctuations in stock prices. The paper presents a portfolio selection model that treats expected return and risk as possibilistic variables, thereby more realistically representing potential stock price values with a small number of changes over time. In addition to returns and risk, the model also incorporates values from financial statements, thus enhancing the solutions obtained. Therefore, optimal portfolios change not only with risk preference but also with changes in the required level of intellectual capital efficiency, liquidity level, or dividend amount. The model is applied to the selection of stock portfolios of Vojvodina companies traded on the Belgrade Stock Exchange. Most of these companies are characterized by low stock liquidity and a small number of price changes, making possibilistic programming ideal for modeling such phenomena.

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